A dynamical model for the penumbral fine structure and the Evershed effect in sunspots

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ABSTRACT

Relying on the assumption that the interchange convection of magnetic flux tubes is the physical cause for the existence of sunspot penumbrae, we propose a model in which the dynamical evolution of a thin magnetic flux tube reproduces the Evershed effect and the penumbral fine structure such as bright and dark filaments and penumbral grains.

According to our model, penumbral grains are the manifestation of the footpoints of magnetic flux tubes, along which hot subphotospheric plasma flows upwards with a few km/s. Above the photosphere the hot plasma inside the tube is cooled by radiative losses as it flows horizontally outwards. As long as the flowing plasma is hotter than the surroundings, it constitutes a bright radial filament. The flow confined to a thin elevated channel reaches the temperature equilibrium with the surrounding atmosphere and becomes optically thin near the outer edge of the penumbra. Here, the tube has a height of approximately 100 km above the continuum and the flow velocity reaches up to 14 km/s. Such a flow channel can reproduce the observed signatures of the Evershed effect.

Subject headings: Sun – sunspots – sun: magnetic fields – MHD

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1. Introduction

A sunspot penumbra reveals quite a number of phenomena that are not yet fully understood. In high resolution images, high intensity contrasts that evolve dynamically are observed: Radially elongated bright and dark filaments, having a width $< 0.35''$ (Muller 1973b), and penumbral grains migrating towards the umbra. Muller describes penumbral grains as having a *comet-like* structure: a bright *coma* and a somewhat thinner and dimmer tail pointing radially outwards. There is observational evidence that bright filaments consist of a few penumbral grains, that are radially aligned (Muller 1973b; Soltau 1982; see also Muller 1992).

Analyzing the Stokes-I profiles of photospheric absorption lines within the penumbra, Evershed (1909) measured a shift of the line core, indicating a radial and horizontal outflow of mass. Additional characteristic features of the Evershed effect are that penumbral Stokes-I profiles are asymmetric, and that the line shift is most intense in dark filaments (e.g. Beckers & Schröter 1969; Degenhardt 1993; Wiehr 1995). Stationary siphon-flow models (Meyer & Schmidt 1968) seem promising in order to explain the Evershed effect (Thomas 1988; Degenhardt 1991; Thomas & Montesinos 1993). However, these models depend on the existence of strong magnetic flux concentrations near the outer edge of the penumbra, in order to create a gas pressure gradient between two footpoints of an arched magnetic flux tube that accelerates the plasma. Here, we want to present a first step towards a dynamical model, in which the gas pressure gradient that accelerates the plasma builds up locally within the penumbra.

It has been pointed out by Schmidt (1987, see also Schmidt 1991) that the penumbra containing a significant fraction of the sunspot's magnetic flux has to be deep in the sense that it extends many scale heights beneath the photosphere. In order to explain the surplus brightness of the penumbra as compared to the umbra, Jahn & Schmidt (1994, hereafter JS) proposed the concept of interchange convection, in which heat that leaks through the magnetopause is distributed horizontally within the penumbra by an interchange of magnetic flux tubes. Parameterizing the effects of interchange convection, JS constructed a tripartite model that consists of three distinct hydrostatic stratifications for the umbra, the penumbra, and the quiet sun, respectively. Two current sheets, i.e., the peripatopause separating the umbra from the penumbra and the magnetopause separating the penumbra from the quiet sun, create a current-free magnetostatic field that is consistent with the observed geometry of the magnetic field, and which satisfies total pressure equilibrium horizontally across the current sheets.

Studying the consequences of interchange convection, we perform numerical simulations which describe the dynamical evolution of a thin magnetic flux tube embedded in a tripartite sunspot model of JS (sec. 2; for some preliminary results see also Jahn et al. 1996, and Schlichenmaier et al. 1997a). Our numerical experiments are compared with observed penumbral phenomena in section 3. We argue that our dynamical model can consistently explain the appearance of bright and dark filaments, penumbral grains, and the Evershed effect, and conclude (sec. 4) that interchange convection offers a promising concept towards an understanding of the penumbra.

2. The evolution of a thin magnetic flux tube within the penumbra

We assume that the penumbral magnetic flux fragments into magnetic flux tubes. These flux tubes take part in the process of interchange convection, and evolve dynamically as physical entities within the penumbra. Thus, the penumbra consists of an ensemble of magnetic flux tubes. As a natural first step towards an understanding of such a penumbra, we study the dynamics of one single flux tube that is embedded in a tripartite sunspot model of JS as a pregiven background.

The dynamical evolution of the magnetic flux tube is described by taking advantage of the thin flux tube approximation (Defouw 1976; Moreno–Insertis 1986). Thus, we neglect magnetic diffusivity and assume that any physical variable does not change across the tube, but can only vary along the tube. We are left with a one-dimensional tube evolving in a two-dimensional background. As the independent variable we use the integrated mass, a, along the tube, and use convective time derivatives to track individual mass elements. It is assumed that the tube instantaneously reaches the total pressure equilibrium with the background:

$$p(a,t) + \frac{B^2(a,t)}{8\pi} = p_{\mathrm{b}}(a,t) + \frac{B_{\mathrm{b}}^2(a,t)}{8\pi} \quad . \tag{1}$$

Here, p and B denote the gas pressure and the magnetic field strength, respectively, and the index b marks local background variables. The equation of state takes into account the partial ionization of hydrogen and helium. The radiative heat exchange of the tube with the background is described using the relaxation time approach of Spiegel (1957). Details of the mathematical and numerical description are given in Schlichenmaier et al. (1997b; see also Schlichenmaier 1997).

Figure 1a shows the initial configuration of our numerical experiment. The tube, having a magnetic flux

$\phi = 2 \cdot 10^{16}$ Mx, lies along the magnetopause, so that part of the tube is in contact with the underlying hotter quiet sun. As the lower boundary condition, the lowest mass element of the tube is held fixed at a depth of $z = -15$ Mm and a radial distance from sunspot center of $x = 5$ Mm, i.e. at the local position of the magnetopause. At the upper end of the tube, initially at $x = 25$ Mm and $z = 400$ km, we choose free boundary conditions: The time derivative of the tangent vector with the modulus length per mass increment is extrapolated and the total force at the last interior grid point is transfered to the outermost grid point. Figure 1 shows the peripatopause, the magnetopause, and the shape of the tube (with its diameter scaled by a factor 12 for better visibility). The photosphere of the quiet sun ($\tau = 2/3$ level) corresponds to $z = 0$ km. The horizontal line at $z = -150$ km in the penumbra represents the Wilson-depressed photosphere of the penumbra. Figure 1a also shows the Wilson depression of the umbra at $z = -470$ km. In Figure 1b and 1c subsequent stages of the evolution of the tube are shown. The shading represents the flow speed along the tube. The background shading corresponds to vanishing velocity and any darker shading to a velocity that points upwards (increasing z) and outwards (increasing x).

The part of the tube that is in contact with the underlying hotter quiet sun is heated by radiation, expands, gets less dense than the surroundings, and rises due to buoyancy. Below the photosphere, the ascending part of the tube is accelerated, because radiative heat exchange is negligible (i.e., the tube rises adiabatically) and the stratification is superadiabatic (i.e., convectively unstable). Above the photosphere the tube ceases to rise, since the tube's density increases due to radiative losses and due to the stabilizing background stratification. The rate at which the tube emerges is dominated by the magnitude of superadiabaticity in the background stratification, which has its maximum a few 100 km below the photosphere. Thus, the tube starts to rise just below the photosphere, and subsequently the upward motion extends to deeper layers. The footpoint of the tube, i.e. the intersection of the tube with the photosphere, migrates towards the umbra as the subphotospheric part of the tube rises (see Fig. 1b).

The forces acting perpendicular to the tube are buoyancy, $\hat{\mathbf{n}} \cdot \mathbf{g}(\rho - \rho_{\mathrm{b}})$, magnetic tension, $\kappa B^2/4\pi$, and the gradient of the magnetic pressure in the background, $-\hat{\mathbf{n}} \cdot \nabla B_{\mathrm{b}}^2/8\pi$ (ρ and B are density and magnetic field strength, respectively, $\mathbf{g}$ is the gravity at the solar surface, κ measures the tube's curvature, and $\hat{\mathbf{n}}$ denotes the unit vector perpendicular to the tube). Since the magnetic pressure decreases upwards inside a sunspot, the tube is pushed upwards. Approximately 100 km above the penumbral photosphere the tube finds an equilibrium in a form of a horizontal channel (with vanishing curvature force) in which the anti-buoyancy force (the tube's density is 10% larger than the background density) is balanced by the gradient of the background's magnetic pressure directed upwards.

The tube stays in total pressure equilibrium with the background as it rises (cf. eq. 1). The magnetic background pressure decreases very little relative to the background gas pressure, which decreases exponentially with a scale height of the order of 100 km. This implies that below the photosphere the total pressure decreases according to the gas pressure scale height, since $\beta = 4\pi p/B^2 \gg 1$. Thus, the plasma within the tube expands as it rises, and because magnetic flux is conserved along the tube, the magnetic pressure within the tube decreases. From equation (1) it follows that the gas pressure inside the tube must become larger than the gas pressure of the background, so that a surplus gas pressure builds up within the tube as it rises.

At the outer edge of the penumbra, above the photosphere, the tube does not rise and the gas pressure inside the tube does not change there. The part of the tube that has risen to the photosphere has a surplus gas pressure. Since the external pressure above the photosphere is almost constant on horizontal planes a gas pressure gradient, $\partial p/\partial a$, builds up along the horizontal part of the tube. In result, the plasma is accelerated outwards, since gravity, $\hat{\mathbf{t}} \cdot \mathbf{g}\rho$, cannot counteract the gas pressure gradient in an horizontal tube. That means that as the tube rises through the subphotospheric penumbra, a longitudinal flow develops pointing upwards below the photosphere and horizontally outwards above the photosphere.

As the inclination of the subphotospheric part of the tube becomes steeper, the buoyancy diminishes and the footpoint stops migrating inwards. Now, one would expect that magnetic tension pulls the tube back down again, but at the turning point the longitudinal flow exerts a centrifugal force that can counteract the magnetic tension and prevent the tube from sinking down. Figure 1c shows the tube after 5400 s. One can see that the footpoint of the tube, which started at $x = 13.5$ Mm, has migrated inwards to $x \approx 8.5$ Mm. For the further evolution of the tube, the reader is referred to Schlichenmaier et al. (1997b).

3. Comparison with observations

With this simulation at hand we can offer an explanation for the penumbral grains, bright and dark filaments, and the Evershed effect. In Figure 2 a magnified snapshot of the tube, which is elevated some 100 km above the penumbral photosphere, is shown for

$t = 5400$ s. Here the gray scale represents the temperature of the tube and of the background. The length of the arrows in the tube corresponds to the longitudinal flow speed. The tube's diameter is magnified by a factor of 6.

Penumbral grains: The upflow in the subphotospheric tube brings up hot plasma, which makes the footpoint look bright. At the footpoint the upflow velocity amounts to 3 km/s. A penumbral grain can be explained by such a footpoint, because footpoints are hotter and brighter than the surroundings and migrate inwards towards the umbra. Also, due to the high temperature, the optical thickness of the tube that corresponds to the diameter of the tube at the footpoint amounts to $\tau \approx 10^3$, indicating that penumbral grains should be optically thick.

Bright filaments: From the footpoint, the hot plasma flows outward horizontally and cools gradually due to radiative losses. At some point (in our simulation at $x \approx 10.5$ Mm, see Figure 2) the flow reaches temperature equilibrium with the surroundings.

The horizontal part of the tube that is hotter than the surroundings can be identified as the tail of a penumbral grain. This part is dimmer than the footpoint because it is cooler. It also gets compressed as the plasma loses internal energy, and the tube becomes thinner. Our model then suggests that bright filaments are the tails of penumbral grains. However, it may also be that an observed bright filament consists of a few penumbral grains representing some distinct flux tubes that are radially aligned.

Dark filaments: As long as the plasma is hotter than the surrounding atmosphere, its opacity is high due to the temperature dependence of the H^- opacity, $\tilde{\kappa}(H^-) \propto T^9$. Hence, the tube is optically thick as long as it is hot. When the flow within the tube reaches temperature equilibrium it becomes as transparent ($\tau \approx 10^{-1}$) as the atmosphere at a similar height. In our simulated tube, the length of the bright filament would measure ≈ 2000 km with a thickness of less than 50 km. The length of a bright filament depends on how fast the plasma is cooled. A thinner tube would have a smaller optical thickness and would be cooled more efficiently, forming a shorter bright filament.

According to our model, dark filaments do not exist *per se*, but are caused by optically thick bright filaments that partially cover the darker penumbral photosphere. The spacing between two adjacent bright filaments that are radially elongated appears as a dark filament. This idea is consistent with the statement of Muller (1973a, 1973b), that within the penumbra, bright features *show up* against a dark background.

Evershed effect: Once the plasma reaches temperature equilibrium with its surroundings, it becomes optically thin, i.e., $\tau \approx 10^{-1}$. Hence, as can be seen in Figure 2, the tube is transparent between $x \approx 10.5$ Mm and the outer edge of the penumbra. There, a line-of-sight that crosses the tube reaches optical depth $\tau = 2/3$ near the photosphere of the model atmosphere at $z \approx -150$ km. Thus, the flux tube appears as a flow channel, being transparent, thin, and elevated with respect to the photosphere. Between the footpoint and the point of temperature equilibrium the gas pressure inside the tube decreases due to radiative losses. By this gas pressure gradient, the flow is accelerated from 3 km/s at the footpoint up to a supercritical speed of 14 km/s near the outer edge of the penumbra.

In order to reproduce the characteristic features of the Evershed effect, namely that penumbral line profiles are shifted and asymmetric, such that the line wing is more shifted than the line core (see, e.g., Degenhardt 1993), we propose the following geometry. Assume that the line core originates mainly some 200 km above the penumbral photosphere, whereas the main contributions for the line wing (20% line depression) stems from, say 100 km above continuum. A flow channel at and below 100 km above continuum would then only affect and shift the line wing, whereas the line core would be mostly unaffected. For such a configuration, our model reproduces observed line profiles, as, e.g., the line profile of Fe I 709.0 nm, which is shown in Figure 1b of Degenhardt (1993). In this context, we mention that Rimmele (1995) and Stanchfield et al. (1997) interpret their observations as being caused by thin elevated flow channels.

Wiehr (1995) deduces a flow velocity of ≥ 5 km/s by assuming that the line asymmetry is caused by a spatially unresolved line satellite caused by a flow channel. If one further takes into account that a typical contribution function has a full width at half maximum of more than 100 km and that our simulated tube has a diameter of less than 50 km, a local flow velocity exceeding 10 km/s is still consistent with observation.

4. Discussion and conclusion

Relying on the concept of interchange convection, we study the dynamics of one thin magnetic flux tube that evolves in the penumbra. The results of this paper show that penumbral grains, bright filaments, and the Evershed effect may be caused by the dynamical evolution of magnetic flux tubes that are embedded in the penumbra. According to our model, bright penumbral phenomena are the consequence of an upflow of

hot subphotospheric plasma that is channeled by magnetic flux tubes. Simultaneously, one can understand the Evershed flow as being the extension of that upflow. After the flow is cooled due to radiation losses, the tube becomes transparent. The asymmetry in the line profile appears because the flow channel affects mostly the line wing of a photospheric absorption line.

We have studied the evolution of one particular flux tube that lies along the magnetopause initially. Beyong the visible edge of the penumbra the magnetopause ascends toward the magnetic canopy in a convectively stable atmosphere. In our simulation, the flow follows the magnetopause. Sofar, we did not study the case of a downstream leg near the outer edge of the penumbra.

Our model suggests that the concept of interchange convection seems very promising. We can propose a coherent picture of the physical structure of the penumbra based on the results presented here. Furthermore, the concept of interchange convection seems to explain the brightness of the penumbra as compared to the umbra. As we show in this paper, an upflow along magnetic flux tubes develops, transporting hot subphotospheric plasma up to the photosphere. However, for an understanding of the behavior of a complete ensemble of penumbral flux tubes, different initial conditions have to be studied.

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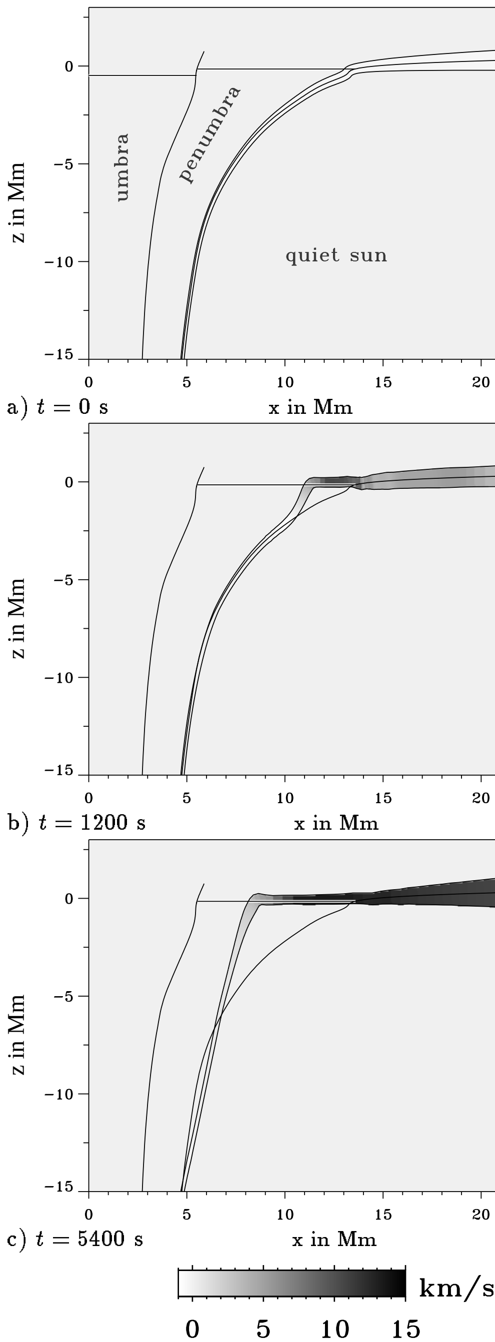
Fig. 1.— The shape of the flux tube is shown for $t = 0$, 1200, and 5400 s. Umbra and penumbra are separated by the peripatopause, penumbra and quiet sun by the magnetopause. The horizontal line within the penumbra represents the photosphere of the penumbra. The gray scale shows the flow speed along the tube. Note that the tube's diameter is magnified by a factor 12 for better visibility.

Fig. 2.— At $t = 5400$ s the tube near the solar surface is shown. The horizontal line at $z = -150$ km represents the photosphere of the penumbra. The gray scale represents the temperature of the tube and of its surroundings. On the left side the cooler umbra and on the right side the hotter quiet sun are visible. The tube's diameter is magnified by a factor 6. The arrows represent longitudinal flow speeds. In the lower left corner an arrow representing 5 km/s is drawn.


umbra
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quiet sun
z in Mm
x in Mm
a) $t = 0$ s
b) $t = 1200$ s
c) $t = 5400$ s
km/s
0
5
10
15